10 MA4

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PERFECT ☺

1 a. Principle of mathematical induction

~~0,5~~ Let S_n be a statement about a positive

0,5 integer n. Suppose that

1. S_1 is true
2. S_{k+1} is true whenever S_k is true

Then the statement S_n holds for every positive integer n.

b. S_n $1+5+9+\dots+(4n-3) = 2n^2-n$

~~1,5~~ Given $n \geq 1$ and n is a natural number

1,3 Proof by mathematical induction.

<u>Base step</u> take $n=1$

$(4 \cdot 1)-3 = (2 \cdot 1^2)-1$

$4-3 = 2-1$

$1 = 1$

This statement holds for $n=1$.

<u>Induction step</u> Assume S_n holds for $n=k$. Show that S_n also holds for $n=k+1$

S_k. $1+5+9+\dots+(4k-3) = 2k^2-k$

S_{k+1} $\underbrace{1+5+9+\dots+(4k-3)}_{\text{assumption can be written as } 2k^2-k} + (4(k+1)-3) = 2(k+1)^2-(k+1)$

S_{k+1}. $2k^2-k+(4(k+1)-3) = 2(k+1)^2-(k+1)$

$2k^2-k+4k+4-3 = 2(k^2+2k+1)-k-1$

$2k^2+3k+1 = 2k^2+4k+2-k-1$

$2k^2+3k+1 = 2k^2+3k+1$

Since the left and right side are both $2k^2+3k+1$, this statement S_n holds for $n=k+1$. Since this statement holds for $k+1$, I proved, by mathematical induction, that this statement holds for every positive integer n. Q.E.D.

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(2) $S_n \quad 9 \mid 4^{3n} - 1$

(1.8) Given: n is a positive integer.

Proof by mathematical induction:

<u>Base step</u>: take $n = 1$

$4^{3 \cdot 1} - 1 =$

$4^3 - 1 =$

$64 - 1 = 63$

9 divides 63 because $7 \cdot 9 = 63$.

This statement S_n holds for $n = 1$.

<u>Induction step</u>: Assume the statement S_n holds for $n = k$. Show this statement also holds for $n = k+1$.

$S_k: \quad 9 \mid 4^{3k} - 1$

$S_{k+1}: \quad 9 \mid 4^{3(k+1)} - 1$

$4^{3(k+1)} - 1 =$

$4^{3k+3} - 1 =$

$4^3 \cdot 4^{3k} - 1 =$

$64 \cdot 4^{3k} - 1 =$

$64(4^{3k} - 1) + 63$

$64(4^{3k} - 1)$ is a multiple of my assumption. It is 64 times my assumption. Since I assumed my assumption is divisible by 9, ~~may~~ the multiple of this assumption is also divisible by 9.

63 can be divided by 9 because $7 \cdot 9$ is 63.

Since $9 \mid 64(4^{3k} - 1) + 63$, this statement is true for $n = k+1$. Since this statement is true for $n = k+1$, the statement S_n holds for every positive integer n, proved by mathematical induction. Q.E.D

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(3)
(1,8)

$z^4 = -i$

first rewrite z^4 and $-i$ to Euler notation

$z^4 = r^4 \ e^{i4\theta}$

$-i = 1 \cdot e^{i(1\frac{1}{2}\pi + k \cdot 2\pi)}$ $\qquad k \in \mathbb{Z}$

$r^4 = 1$

$r = 1 \quad \vee \quad \cancel{r = -1}$ (not a solution because r has to be positive)

$e^{i4\theta} = e^{i(1\frac{1}{2}\pi + k\cdot 2\pi)}$

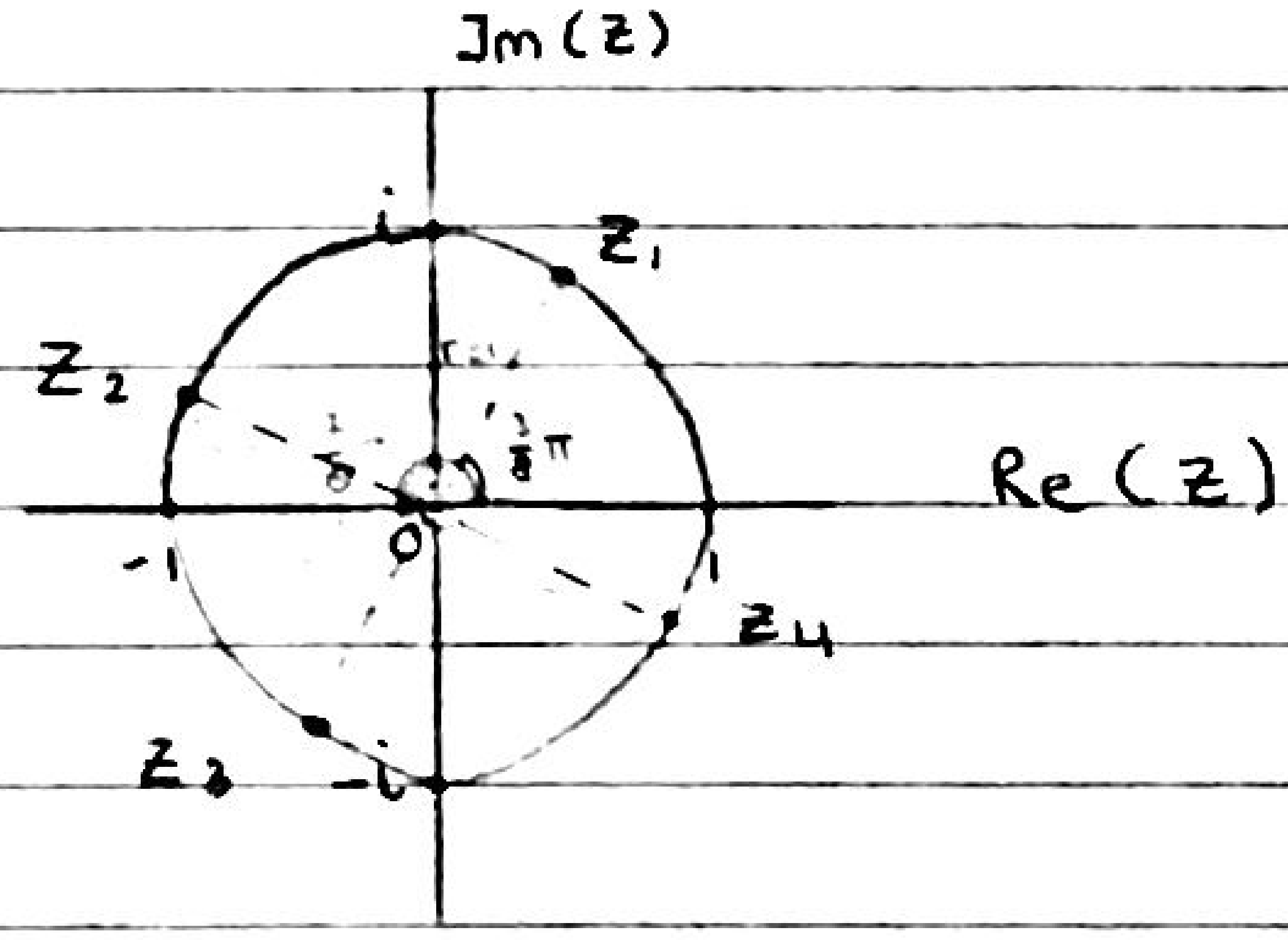
$4\theta = 1\frac{1}{2}\pi + k \cdot 2\pi$

$\theta = \frac{3}{8}\pi + k \ \frac{1}{2}\pi$

4 solutions

1. $z_1 = 1 \ e^{i(\frac{3}{8}\pi)} = e^{i(\frac{3}{8}\pi)}$
2. $z_2 = 1 \ e^{i(\frac{7}{8}\pi)} = e^{i(\frac{7}{8}\pi)}$
3. $z_3 = 1 \ e^{i(\frac{11}{8}\pi)} = e^{i(1\frac{3}{8}\pi)}$
4. $z_4 = 1 \cdot e^{i(\frac{15}{8}\pi)} = e^{i(1\frac{7}{8}\pi)}$

sketch in complex plane:



(4)
(1,8)

$e^{-z} = \frac{e^{z}}{2}$

$2e^{-z} = e^{z}$

$\frac{2e^{-z}}{e^{z}} = 1$

$2e^{-2z} = 1$

$2e^{-2(a+bi)} = 1$

$2e^{-2a-2bi} = 1$

$e^{-2a-2bi} = \frac{1}{2}$

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$e^{-2a-2bi} = \frac{1}{2}$

$e^{-2a} e^{-2bi} = \frac{1}{2} \cdot e^{(k \cdot 2\pi)i}$ $\qquad k \in \mathbb{Z}$

$e^{-2a} = \frac{1}{2}$

$-2a = \ln\left|\frac{1}{2}\right|$

$a = -\frac{1}{2}\ln\left|\frac{1}{2}\right|$

$e^{-2bi} = e^{(k \cdot 2\pi)i}$

$-2b = k \cdot 2\pi$

$b = -k \cdot \pi$

$z = -\frac{1}{2}\ln\left|\frac{1}{2}\right| - (k \cdot \pi)i = \frac{1}{2}\ln(2) + (k\pi i)$

5

1,8

$\lim_{x \to 1} x^2 = 1$

$\forall \varepsilon > 0 \ \exists \delta > 0 \quad 0 < |x-1| < \delta \Rightarrow |x^2-1| < \varepsilon$

$|x^2-1| < \varepsilon$

$|(x-1)(x+1)| < \varepsilon$

$\delta > x-1$

assume $\delta = 1$

$0 < |x-1| < \delta$

$0 < |x-1| < 1$

$1 < |x| < 2$

$|x| < 2$

$|x+1| < 3$

$\delta = \{\min(1, \frac{\varepsilon}{3})\}$

So $\lim_{x \to 1} x^2 = 1$ proved by ε, δ definition

of a limit. Q.E.D

1	2	3	4	5	Σ+1
1,8	1,8	1,8	1,8	1,8	10